Universidad Europea

Line Integrals

- Calculate the arc length of the following curves:
  - $(2\cos(t), \sin(t), t)$, $t \in [0, 2\pi]$.
  - $\left(t+1, \frac{2\sqrt{2}}{3}t^{3/2}+7, \frac{1}{2}t^2\right)$, $1 \leq t \leq 2$.
  - (t, t^2, t^3), $0 \leq t \leq 2$.
- Let $c(t) = (t, t\sin(t), t\cos(t))$ be a path describing a curve. Calculate the arc length that connects the points $(0, 0, 0)$ and $(\pi, 0, -\pi)$.
- Show that the trajectory $c(t) = (\cos(t), \sin(t))$ is a flow line of the vector field $F(x, y) = -y\mathbf{i} + x\mathbf{j}$.
- Find the flow lines of the vector field $F(x, y) = (x, y)$.
- Find the flow lines of the vector field $F(x, y) = (a, a)$ with $a \in \mathbb{R} \setminus \{0\}$.
- Find the flow lines of the following vector field $F(x, y, z) = (2x, y-1, z)$.
- Calculate the divergence and the curl of $F = x^2y\mathbf{i} + z\mathbf{j} + xyz\mathbf{k}$.
- If $f, g : A \subseteq \mathbb{R}^3 \to \mathbb{R}$ and $F, G : B \subseteq \mathbb{R}^3 \to \mathbb{R}^3$, prove the following properties.
  - $\nabla(fg) = f\nabla(g) + g\nabla(f)$.
  - $div(F + G) = div(F) + div(G)$.
  - $curl(F + G) = curl(F) + curl(G)$.
  - $div(fF) = fdiv(F) + \nabla(f)$.
- Let $f(x, y, z) = y$ and $c(t) = (0, 0, t)$ with $0 \leq t \leq 1$. Show that the path integral $\int_c f ds = 0$.
- Let $f(x, y, z) = x\cos(z)$ and $c = t\mathbf{i} + t^2\mathbf{j}$, with $t \in [0, 1]$. Calculate the path integral $\int_c f ds$.
- Let $f(x, y, z) = \frac{x+y}{y+z}$ and $c = t\mathbf{i} + \frac{2}{3}t^{3/2}\mathbf{j} + t\mathbf{k}$, with $t \in [1, 2]$. Calculate the path integral $\int_c f ds$.

- Let $F(x, y) = x\mathbf{i} + y\mathbf{j}$ and consider the following two curves $c_1(t) = (t^2, t^2)$ with $-1 \leq t \leq 1$ and $c_2(t) = (2 - t, (2 - t)^2)$ with $1 \leq t \leq 3$. Calculate the following line integrals:

$$\int_{c_1} F ds, \int_{c_2} F ds.$$

- Use the Green's Tehorem to evaluate the integral

$$\int_C y dx - x dy,$$

where C is the boundary of the square $[-1, 1] \times [-1, 1]$ using the opposite direction of the clock holes.

- Verify that in the following examples, the Green's Theorem is verified using D the circle with center $(0, 0)$ with radius R.
  - $P(x, y) = 2y$, $Q(x, y) = x$.
  - $P(x, y) = x + y$, $Q(x, y) = y$.

- Find the area bounded by an arc of the cycloid $x = a(\theta - \sin(\theta))$, $y = a(1 - \cos(\theta))$, where $a > 0$ and $\theta \in [0, 2\pi]$ using the Green's Theorem.